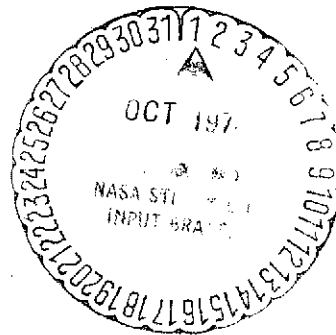


PLANAR NONLINEAR OSCILLATIONS OF THE CENTER OF MASS OF A SOLID WITH
A MAGNETIC DAMPER

K. K. Lavrinovich

Translation of "Ploskiye nelineynnye kolebaniya otnositel'no tsentra mass tverdogo tela s magnitnym dempferom," in: Mekhanika upravlyayemogo dvizheniya i problemy kosmicheskoy dinamiki, Ed. by V. S. Novoselov, Leningrad University Press, Leningrad, 1972, pp. 70-82.



(NASA-TT-F-15811) - PLANAR NONLINEAR
OSCILLATIONS OF THE CENTER OF MASS OF A
SOLID WITH A MAGNETIC DAMPER (Kanner
(Leo) Associates) 20 p HC \$4.00

N74-33144

CSCL 20K G3/23

Unclas
48551

1. Report No. NASA TT F-15811		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle PLANAR NONLINEAR OSCILLATIONS OF THE CENTER OF MASS OF A SOLID WITH A MAGNETIC DAMPER				5. Report Date	
				6. Performing Organization Code	
7. Author(s) K. K. Lavrinovich				8. Performing Organization Report No.	
				10. Work Unit No.	
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063				11. Contract or Grant No. NASW-2481-407	
				13. Type of Report and Period Covered Translation	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D. C. 20546				14. Sponsoring Agency Code	
15. Supplementary Notes Translation of "Ploskiye nelineynnye kolebaniya otnositel'no tsentra mass tverdogo tela s magnitnym dempferom," in: Mekhanika uprallyayemogo dvizheniya i problemy kosmicheskoy dinamiki, Leningrad, University Press, Leningrad, 1972, pp. 70-82.					
16. Abstract This study examines nonlinear planar oscillations of a solid body with respect to the center of mass under the following assumptions: (1) the center of mass of this body moves in a circular trajectory by gravity around another body having a central newtonian gravitational field; (2) the attracting body has a magnetic field; (3) the moving body contains a damper (magnetic)--damping momentum is proportional to the first power of angular velocity of motion of the magnetized globe with respect to the damper; (4) the moving body is affected by external excitation momentum represented as a Fourier series in terms of powers of orbit frequency.					
17. Key Words (Selected by Author(s))				18. Distribution Statement Unclassified-Unlimited	
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 181i	
				22. Price	

PLANAR NONLINEAR OSCILLATIONS OF THE CENTER OF MASS
OF A SOLID WITH A MAGNETIC DAMPER

K. K. Lavrinovich

②

1. The proposed study examines nonlinear planar oscillations of a solid body with respect to the center of mass under the following assumptions:

/70

1) the center of mass of this body moves in a circular trajectory under the influence of gravitational forces around a second body which has a central newtonian gravitational field; the mass of the moving body is negligibly small versus the mass of the attracting body.

2) the attracting body also has a magnetic field which can be represented as a dipole model. The plane of trajectory of the moving body passes through the poles of the magnetic field;

/71

3) on the moving body is set a magnetic damper, which is a magnetized metal sphere immersed in a chamber containing a viscous fluid. The damping effect is achieved due to the viscous friction between the fluid, the walls of the chamber and the magnetized globe which, with the oscillations of the body, tends to remain immobile with respect to the local vector of the force line of the external magnetic field. The damping momentum is proportional to the first power of angular velocity of motion of the magnetized globe with respect to the damper body. This damping device is described in study [1] among others;

4) the moving body in the plane of trajectory is affected by the external excitation momentum which can be represented as a Fourier series in terms of powers of orbital frequency.

Supposition 1) permits us to examine oscillations of the solid body with respect to the center of mass independently of trajectory of motion.

Let us connected rigidly with the body a rectangular system of coordinates (x, y, z) , whose center coincides with the center of mass of the body, and the axes are directed along the main central axes of inertia, where the x axis corresponds to the largest axis of the ellipsoid of inertia; the y axis to the middle; the z axis to the smallest. Let us also introduce an orbital system of coordinates (x_1, y_1, z_1) which has a common center with system (x, y, z) and the following direction of axes: axis x_1 coincides with the direction of the radius-vector of the center of mass with respect to the center of gravity; axis y_1 --with the direction of the positive transversal, axis z_1 is binormal to the plane of trajectory. If like axes of both systems are collinear, the body is in one of the four positions of stable equilibrium. Equations describing body oscillations in the plane of trajectory with respect to any of these positions of equilibrium, with accuracy to within the notations, coincide for determinacy we will consider oscillations with respect to the position of equilibrium which is typified by a coincidence of like axes. As coordinates let us select the angle θ , read from the positive direction of axis x_1 to the positive direction of axis x . Without going into the details of the derivation, which can be found in study [2], let us write the equation of plane oscillations of the body in the form

$$\ddot{\theta} + 3\omega_0^2 \frac{I_y - I_x}{I_z} \sin \theta \cos \theta = -\frac{k_d}{I_z} (2\omega_0 F^2 - \dot{\theta}) + \frac{M_z}{I_z} \quad (1)$$

Here the notations are used: ω_0 --angular velocity of orbital motion; I_x, I_y, I_z --moments of inertia of the body with respect to the corresponding axes; k_d --the coefficient of damping, equal to the coefficient of proportionality between the angular velocity

of motion of the magnetized globe with respect to the body of the damper and the damping moment;

$$F^2 = \frac{1}{1+3\sin^2 \omega_0 t} = \frac{1}{2} + \sum_{m=1}^{\infty} \frac{1}{3^m} \cos 2m \omega_0 t, \quad (2)$$

M_z -- external excitation moment.

The dot signifies differentiation with respect to time t .
Let us move on in equation (1) to the argument $u = \omega_0 t$ and make the substitution $\theta = \frac{\vartheta}{2}$:

$$\vartheta'' + \alpha^2 \sin \vartheta = \varepsilon (4F^2 - \vartheta' + M_z^*), \quad (3)$$

where it is designated that

$$\alpha^2 = 3 \frac{I_y - I_x}{I_z}, \quad \varepsilon = \frac{k_d}{I_z \omega_0},$$

$$M_z^* = \frac{2M_z}{\omega_0 k_d} = c_0 + \sum_{n=1}^{\infty} (c_n \sin nu + d_n \cos nu); \quad (4)$$

the prime signifies differentiation with respect to u . In assuming that k_d is small, we can consider (3) as a nonlinear differential equation of the second order, containing small parameter ε . With respect to the coefficients c_n , d_n we will assume that they are sufficiently small and decrease in absolute magnitude with an increase in n .

2. The approximate solution of equation (3) will be sought using the method of averaging, employing the property of smallness of parameter ε .

When $\varepsilon = 0$ we derive the homogeneous nonlinear equation

$$\vartheta'' + \alpha^2 \sin \vartheta = 0, \quad (5)$$

whose solution for the case of oscillations will be the function

$$\theta = 2 \arcsin [k \cdot \operatorname{sn} [a(u + u_0), k]]. \quad (6)$$

This solution depends on two arbitrary constants k and u_0 , the modulus k of the elliptical function having the sense of a sine of the amplitude of oscillations of angle θ . The solution of (6) has in u a period $T = (4K)/a$ and a corresponding round frequency

$\omega(k) = (\pi a)/(2K)$, where $K(k) = \int_0^{\pi/2} \frac{d\psi}{\sqrt{1-k^2 \sin^2 \psi}}$ --the complete elliptical integral of the first order. With the admission of a new variable ϕ in the formula $\phi = \omega(k)(u + u_0)$, we bring solution (6) to the period 2π

$$Q = 2 \arcsin \left[k \cdot \operatorname{sn} \left(\frac{2K}{\pi} \varphi, k \right) \right]. \quad (7)$$

The solution of equation (3) will be sought in the form of (7), /73 considering ϕ and k as unknown functions of u and assuming also that

$$\frac{dQ}{du} = \omega(k) Q_\varphi, \quad (8)$$

where the lower index signifies the partial derivative with respect to ϕ . Let us substitute (7) in (3) and (8) and solve the derived system with respect to the derivative functions $k(u)$ and $\phi(u)$

$$\begin{cases} \frac{dk}{du} = \frac{\varepsilon (4F^2 - P + M_s^*) Q_\varphi}{P_k Q_\varphi - P_\varphi Q_k} \\ \frac{d\varphi}{du} = \omega(k) - \frac{\varepsilon (4F^2 - P + M_s^*) Q_k}{P_k Q_\varphi - P_\varphi Q_k} \end{cases} \quad (9)$$

Here we have adopted the notation $\frac{dQ}{du} = P$. A similar system is examined in study [3]. There is it proven that the denominator

on the right sides of (9) does not depend on ϕ and that it is equal to the derivative with respect to k

$$I'(k) = P_k Q_\phi - P_\phi Q_k \quad (10)$$

from the integral of action of equation (3)

$$I(k) = \frac{\omega(k)}{2\pi} \int_0^{2\pi} Q_\phi^2 d\phi \quad (11)$$

Let us re-write (9) taking into account (10)

$$\begin{cases} \frac{dk}{du} = \frac{\varepsilon}{I'(k)} (4F^2 - P + M_s^*) Q_\phi, \\ \frac{d\phi}{du} = \omega(k) - \frac{\varepsilon}{I'(k)} (4F^2 - P + M_s^*) Q_k. \end{cases} \quad (12)$$

From (12) it is clear that k is a slow variable, while the derivative $\frac{d\phi}{du}$ differs from $\omega(k)$ by a quantity on the order of ε . Moreover, the right sides of (12) are periodic with respect to ϕ and u with a period of 2π . Consequently, we can derive a solution of system (12) with an accuracy on the order of ε exclusively, averaging the right sides with respect to u and ϕ .

3. Let us assume that at the onset of motion described by equation (3), frequency $\omega(k)$ of inherent oscillations is not close to some fraction r/s of the frequency of perturbation (r, s -- natural mutually simple integers). In this context, it is admissible to average the right sides of (12) with respect to u and ϕ as independent variables: /74

$$\begin{cases} \frac{dk}{du} = \frac{\varepsilon}{I'(k)} \cdot \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} (4F^2 - P + M_s^*) Q_\phi du d\phi, \\ \frac{d\phi}{du} = \omega(k) - \frac{\varepsilon}{I'(k)} \cdot \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} (4F^2 - P + M_s^*) Q_k du d\phi. \end{cases}$$

Let us note that functions $Q(\phi, k)$ and $Q_k(\phi, k)$ are non-even with respect to the argument ϕ , while function $Q_\phi(\phi, k)$ is even and all named functions have a period of 2π with respect to ϕ . Considering these properties of the subintegral functions, and also bearing in mind (2) and (4), we find that,

$$\begin{cases} \frac{dk}{du} = -\frac{\varepsilon}{I'(k)} \cdot \frac{\omega(k)}{2\pi} \int_0^{2\pi} Q_\varphi^2 d\varphi, \\ \frac{d\varphi}{du} = \omega(k). \end{cases} \quad (13)$$

The integrals from the terms containing F^2 and M_z^* are equal to zero, i.e., in this nonresonant case, the velocity of attenuation of oscillatory amplitude is not a function, in the first approximation, neither of external perturbations M_z^* nor of periodic perturbations experienced by the body from interaction of the damper core with the external magnetic field.

On the right side of the first equation of (13) stands the integral of action (11). Let us differentiate (7) with respect to ϕ

$$Q_\varphi = \frac{4kK}{\pi} \operatorname{cn}\left(\frac{2K}{\pi}\varphi, k\right), \quad (14)$$

let us substitute (14) into (11) and integrate (11):

$$I(k) = \frac{8\pi}{\pi} (E - k'^2 K). \quad (15)$$

Let us calculate here $I'(k)$:

$$I'(k) = \frac{8\pi}{\pi} kK. \quad (16)$$

Here $E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \psi} d\psi$ ---the complete elliptical integral

of the second order; k' and k are connected by the relationship $k'^2 = 1 - k^2$. In calculating (14), (15), (16) we employ the laws and formulas cited in [4] and [5]. Substituting (15) and (16) in the first equation of (13), we find that

$$\frac{dk}{du} = -\frac{\varepsilon}{k'K} (E - k'^2 K). \quad (17)$$

Because all coefficients on the right side of (17) are positive, /75 then $\frac{dk}{du} < 0$, i.e., in the nonresonant case the amplitude decreases with time and we have an attenuating oscillatory process. To integrate equation (17) is not exactly possible due to the complex relationship of the right side as a function of k . The relationship $k(u)$ can be derived as a result of a numerical solution of equation (17) on computer. Let us note that formula (17) is valid to within ε accuracy in the interval of change of the argument on the order of $\frac{T}{\varepsilon}$.

4. We can not, however, be assured that in all cases oscillation will develop as an attenuating process. Indeed, the initial value of inherent frequency $\omega(k)$ does not remain constant in the event of attenuating oscillations, it increases in proportion to a decrease in k . Consequently, at some moment our assumption that $\omega(k) \neq r/c$ will be untrue and at some frequency, equal or close to this value, oscillations can 'get stuck,' i.e., become stationary.

Let us define the value of amplitude and phase of oscillations in stationary resonant conditions, i.e., when the approximate equality is fulfilled

$$\omega(k) \approx \frac{r}{s}.$$

Study of this case follows the scheme of study [3]. Let us first make several transformations. With allowance for (2) and (4),

let us write

$$4F^2 + M_s^* = (2 + c_0) + \sum_{n=1}^{\infty} (c_n \sin nu + d'_n \cos nu), \quad (18)$$

where

$$d'_n = \begin{cases} d_n, & \text{if } n \neq 2m, \\ d_n + \frac{4}{3^{n/2}}, & \text{if } n = 2m. \end{cases}$$

Furthermore, let us introduce a new variable--phase shift κ according to the formula

$$\varphi = \frac{r}{s} (u + \kappa) \quad (19)$$

and move on to κ in equations (12)

$$\begin{cases} \frac{dk}{du} = \frac{\varepsilon}{I'(k)} (4F^2 - P + M_s^*) Q_\varphi, \\ \frac{d\kappa}{du} = \frac{s}{r} \left[\omega(k) - \frac{r}{s} \right] - \frac{\varepsilon}{I'(k)} (4F^2 - P + M_s^*) Q_\kappa. \end{cases} \quad (20)$$

If the different $\omega(k) - r/s$ is sufficiently small (on the order of ε), then κ , as with k , is a slow variable; and in system (20) is permissible averaging with respect to the 'quick' variable ϕ /76
in period equal to the smallest common multiple of the period of inherent oscillations 2π and the period of perturbing function $(2\pi r)/s$, i.e., in period $2\pi r$.

Bearing (8) in mind, we find that

$$\begin{aligned} \frac{dk}{du} = \frac{\varepsilon}{I'(k)} \cdot \frac{1}{2\pi r} \int_0^{2\pi r} & \left\{ (2 + c_0) Q_\varphi + \sum_{n=1}^{\infty} \left[c_n \sin \frac{ns}{r} \varphi \cos nx Q_\varphi - \right. \right. \\ & - c_n \sin nx \cos \frac{ns}{r} \varphi Q_\varphi + d'_n \cos \frac{ns}{r} \varphi \cos nx Q_\varphi + \\ & \left. \left. + d'_n \sin \frac{ns}{r} \varphi \sin nx Q_\varphi \right] - \omega(k) Q_\varphi^2 \right\} d\varphi, \end{aligned}$$

$$\begin{aligned}
\frac{dx}{du} = & \left[\omega(k) - \frac{r}{s} \right] \frac{s}{r} - \frac{\varepsilon}{I'(k)} \cdot \frac{1}{2\pi r} \int_0^{2\pi r} \left\{ (2+c_0) Q_k + \right. \\
& + \sum_{n=1}^{\infty} \left[c_n \sin \frac{ns}{r} \varphi \cos nx Q_k - c_n \sin nx \cos \frac{ns}{r} \varphi Q_k + \right. \\
& \left. \left. + d'_n \cos \frac{ns}{r} \varphi \cos nx Q_k + d'_n \sin \frac{ns}{r} \varphi \sin nx Q_k \right] - \omega(k) Q_\varphi Q_k \right\} d\varphi.
\end{aligned} \tag{21}$$

Because functions Q , Q_k are odd, function Q_ϕ is even and all named functions have a period of 2π , the integrals in (21) from expression Q_ϕ , $\sin \frac{ns}{r} \phi Q_\phi$, Q_k , $\cos \frac{ns}{r} \phi Q_k$, $Q_\phi Q_k$ are equal to zero. The integral of the last term in the first equation of (21) coincides with the integral of action (15). Considering these remarks and noting that

$$b_n = \sqrt{d_n'^2 + c_n^2}, \quad \frac{d'_n}{b_n} = \cos \gamma_n, \quad \frac{c_n}{b_n} = \sin \gamma_n, \tag{22}$$

let us write (21) in a new form

$$\begin{aligned}
\frac{dx}{du} = & \frac{\varepsilon}{I'(k)} \left\{ \sum_{n=1}^{\infty} \left[b_n \cos (nx + \gamma_n) \frac{1}{2\pi r} \int_0^{2\pi r} \cos \frac{ns}{r} \varphi Q_\varphi d\varphi \right] - I(k) \right\}, \\
& \left[\frac{dx}{du} = \frac{s}{r} \left[\omega(k) - \frac{r}{s} \right] - \frac{\varepsilon}{I'(k)} \times \right. \\
& \left. \times \sum_{n=1}^{\infty} \left[b_n \sin (nx + \gamma_n) \frac{1}{2\pi r} \int_0^{2\pi r} \sin \frac{ns}{r} \varphi Q_k d\varphi \right] \right].
\end{aligned} \tag{23}$$

The integral in the first equation will be calculated using the decomposition [4]

$$Q_\varphi = 8 \sum_{i=1}^{\infty} \frac{8q^{i-\frac{1}{2}}}{1+q^{2i-1}} \cos [(2i-1)\varphi],$$

where $q = \exp(-\frac{\pi K'}{K})$ -- a parameter of elliptical Jacobi functions,
 $K' = K(k')$. We then have

$$I_1 = \sum_{n=1}^{\infty} \frac{1}{2\pi r} \int_0^{2\pi r} \left\{ \cos \frac{ns}{r} \varphi \sum_{i=1}^{\infty} \frac{8q^{\frac{i-1}{2}}}{1+q^{2i-1}} \cos[(2i-1)\varphi] \right\} d\varphi.$$

In the expression for I_1 only those terms for which the equality $(ns)/r = 2i - 1$ is fulfilled are not equal to zero. Consequently, let us switch under the summation sign to a common index of summation n

$$\begin{aligned} I_1 &= \sum_{n=\frac{r}{s}(2i-1)}^{\infty} \frac{8q^{\frac{ns}{2r}}}{1+q^{\frac{ns}{r}}} \cdot \frac{1}{2\pi r} \int_0^{2\pi r} \cos^2 \frac{ns}{r} \varphi d\varphi = \\ &= \sum_{n=\frac{r}{s}(2i-1)}^{\infty} \frac{4q^{\frac{ns}{2r}}}{1+q^{\frac{ns}{r}}}, \end{aligned}$$

where i runs through only those natural values for which the index n is also natural for given r, s . Hence it follows, in particular, that with even s , this integral vanishes, resonance does not arise in the system, and we get the aforementioned case of attenuating oscillations.

The second integral in system (23) is calculated, employing integration by parts:

$$\begin{aligned} I_2 &= \frac{1}{2\pi r} \sum_{n=1}^{\infty} \int_0^{2\pi r} \sin \frac{ns}{r} \varphi Q_k d\varphi = \\ &= \frac{1}{2\pi r} \sum_{n=1}^{\infty} \left(-Q_k \frac{r}{ns} \cos \frac{ns}{r} \varphi \right) \Big|_0^{2\pi r} + \\ &+ \frac{1}{2\pi r} \sum_{n=1}^{\infty} \int_0^{2\pi r} \frac{r}{ns} \cos \frac{ns}{r} \varphi \frac{\partial}{\partial k} Q_k d\varphi = \\ &= \frac{1}{2\pi r} \sum_{n=1}^{\infty} \frac{\partial}{\partial k} \left[\frac{r}{ns} \int_0^{2\pi r} \cos \frac{ns}{r} \varphi Q_k d\varphi \right] = \\ &= \sum_{n=1}^{\infty} \frac{r}{ns} \frac{\partial}{\partial k} \left(\frac{4q^{\frac{ns}{2r}}}{1+q^{\frac{ns}{r}}} \right) = \frac{\pi^2}{kk'^2 K^2} \sum_{n=\frac{r}{s}(2i-1)}^{\infty} \frac{q^{\frac{ns}{2r}} \left(1 - q^{\frac{ns}{r}} \right)}{\left(1 + q^{\frac{ns}{r}} \right)^2}. \end{aligned}$$

Here the index i is subordinate to the same condition as the integral I_1 . Therefore, the averaged system (23) acquires the form /78

$$\left\{ \begin{aligned} \frac{dk}{du} &= \frac{4\varepsilon}{I'(k)} \left\{ \sum_{n=\frac{r}{s}(2l-1)}^{\infty} \frac{q^{\frac{ns}{2r}}}{1+q^{\frac{ns}{r}}} b_n \cos(nx+\gamma_n) - \frac{2\alpha}{\pi} (E-k'^2K) \right\}, \\ \frac{dx}{du} &= \frac{s}{r} \left[\omega(k) - \frac{r}{s} \right] - \frac{\varepsilon}{I'(k)} \cdot \frac{\pi^2}{kk'^2K^2} \times \\ &\times \sum_{n=\frac{r}{s}(2l-1)}^{\infty} \frac{q^{\frac{ns}{2r}} \left(1 - q^{\frac{ns}{r}}\right)}{\left(1 + q^{\frac{ns}{r}}\right)^2} b_n \sin(nx+\gamma_n). \end{aligned} \right. \quad (24)$$

Equating the right sides of (24) to zero, we derive an equation for defining k and κ in stationary resonance conditions. In order to make these transcendental equations practically solvable, let us introduce several simplifications. Because $\omega(k) - r/s = 0$ (ε), then without violating the order of accuracy [3], we can derive

$$\omega(k) - \frac{r}{s} = \omega'(k_0)(k - k_0), \quad (25)$$

where k_0 is a value of k which satisfies the precise equality

$$\omega(k_0) = \frac{r}{s}, \quad (26)$$

and the derivative $\omega'(k_0)$ has the form

$$\omega'(k_0) = - \frac{\pi\alpha(E-k'^2K)}{2kk'^2K^2}. \quad (27)$$

Moreover, since the different $k - k_0$ is small, on the right sides of this system we will posit $k = k_0$. We find that

$$\left\{ \begin{aligned} \sum_{n=\frac{r}{s}(2l-1)}^{\infty} \frac{q^{\frac{ns}{2r}}}{1+q^{\frac{ns}{r}}} b_n \cos(nx+\gamma_n) &= \frac{2\alpha}{\pi} (E-k'^2K), \\ k - k_0 &= - \frac{r}{s} \cdot \frac{\varepsilon\pi^2}{4\alpha^2kK(E-k'^2K)} \sum_{n=\frac{r}{s}(2l-1)}^{\infty} \frac{q^{\frac{ns}{2r}} \left(1 - q^{\frac{ns}{r}}\right)}{\left(1 + q^{\frac{ns}{r}}\right)^2} \times \\ &\times b_n \sin(nx+\gamma_n), \end{aligned} \right. \quad (28)$$

where i only accepts natural values, for which n is natural for given r and s . Further we will bear in mind the following concepts. While discussing the resonant case, we assume that the magnitude of frequency of inherent oscillations $\omega(k)$ is close to some irreducible fraction r/s . We will show that from an infinite set of fractions r/s it makes sense to discuss only those whose natural r and s are small. Here three cases are possible:

a) r is great, s is small, i.e., the fraction r/s is greater; by dint of the approximate equality $\omega(k) \sim r/s$, it will contradict the physical sense of the quantity (k) included in the interval $[0, \alpha]$, where $0 \leq \omega \leq \sqrt{3}$;

b) r is small, s is great. In the limiting case we have $r/s \rightarrow 0$, $\omega(k) = \pi\alpha/2K \rightarrow 0$, the latter relationship with fixed α is fulfilled under the condition that $k \rightarrow 1$. Referring to the first equation of (24), we see that

$$\lim_{k \rightarrow 1} \frac{dk}{du} = -\frac{\varepsilon}{I'(k)} \cdot \frac{\delta\alpha}{\pi} (E - k'^2 K),$$

i.e., we have the aforementioned case of attenuating oscillations (the right sides of the latter formula and formula (17) coincide).

c) r is great, s is great. Here let us exclude from the discussion those r and s at which the ratio r/s is greater (case a) or small (case b). At large r and s , the index of summation n in system (24) is also great (the first term unequal to zero below the summation sign has the index $n = r$); consequently, the power of $q^{\frac{ns}{2r}}$ is small (since $q < 1$) and the coefficients b_n are small since c_n, d_n decrease in inverse proportion to n ; thus the terms standing below the summation sign can be ignored and we again derive an equation of the nonresonant case.

Therefore, let us first consider only small r and s where s is odd.

Let us note further that in the case of odd r , the integrals in (23) of terms with even n are equal to zero; with r even, integrals with odd n vanish, i.e., periodic perturbations from the interaction of the damper with the external magnetic field affect the motion only in cases of even r .

Let us study first the possible formation of stationary resonance due to perturbations from the damper, i.e., let us assume r to be even. Since under the assumption that $\omega(k) \sim r/s$ and in general that $0 \leq \omega(k) \leq \sqrt{3}$, then the permissible values of r with given s can be derived from the inequality $0 < r/s < \sqrt{3}$. For example, $s = 1$, no even r exist /80

$s=3$	$r=2, 4$
$s=5$	$r=2, 4, 6, 8$
$s=7$	$r=2, 4, 6, 8, 10, 12$

Let us pause in more detail on the case of minimal values of r and s , i.e., $r = 2$, $s = 3$. Under the summation sign in system (24) will remain terms with the indexes $n = 2, 6, 10, \dots$

$$\sum_{n=\frac{2}{3}(2l-1)}^{\infty} \frac{q^{3/4 n}}{1+q^{3/2 n}} b_n \cos(n\tau + \gamma_n) =$$

$$= \frac{q^{3/2}}{1+q^3} b_2 \cos(2\tau + \gamma_2) + \frac{q^{9/2}}{1+q^9} b_6 \cos(6\tau + \gamma_6) + \dots$$

The quantity $q^{9/2}$, which increases in proportion to k , even when $k = 0.95$, has an order of 0.0004; therefore, in systems (24) and

(28), respectively, we can ignore the summands for which $n \geq 6$. We find that

$$\begin{aligned} \frac{q^{3/2}}{1+q^3} b_2 \cos(2\kappa + \gamma_2) &= \frac{2\alpha}{\pi} (E - k'^2 K), \\ k - k_0 &= -\frac{2}{3} \cdot \frac{\varepsilon \pi^2}{4\alpha^2 k K (E - k'^2 K)} \cdot \frac{q^{3/2} (1-q^3)}{(1+q^3)^2} b_2 \cos(2\kappa + \gamma_2). \end{aligned} \quad (29)$$

The values of k and κ , corresponding to stationary resonance conditions, are found in the following order. From correlation (26), re-written as $\pi\alpha/2K = r/s$, where α , r , s are fixed, with the aid of tables of elliptical integrals (e.g., [6], [7]), we find k_0 ; then we substitute the found k_0 in the first equation of (29) and from it define the pair of resonance values $2\kappa + \gamma_2$, which differ in signs. With known k_0 , κ from the second equation (29) we derive the corresponding pair of values of k . The absence of a solution of system (29) indicates the impossibility of resonance conditions. The first equation has a solution if $|\cos(2\kappa + \gamma_2)| \leq 1$, i.e., if

$$\frac{\frac{2\alpha}{\pi} (E - k'^2 K)}{b_2 \frac{q^{3/2}}{1+q^3}} \leq 1. \quad (30)$$

Numerical analysis of inequality (30) for $0.05 \leq k \leq 0.95$ and $2/3 < \alpha < \sqrt{3}$ shows that it is satisfied only at values of the coefficient b_2 which contradict the supposition of its smallness (e.g., at $k_0 = 0.50$, $b_2 > 33$; at $k_0 = 0.90$, $b_2 > 8$, whereas the value of b_2 having actual meaning has an order of one). Therefore, when $r/s = 2/3$, stationary resonance conditions do not arise in the system. /81

With an increase of r or s , the coefficient $1 + \frac{q^{ns/2r}}{q^{ns/r}} b_n$ decreases: in the first case, due to an increase in the number n in the coefficient b_n ; in the second case--due to an increase in the

14

exponent $ns/2r$. Consequently, with other possible combinations of even r and odd s for the existence of a solution of system (28), it is necessary that the coefficients b_n be still greater than where $r/s = 2/3$. Since the proposed values of b_n are much less, we come to the conclusion that the damper can not evoke stationary resonance conditions in the body's oscillatory motion.

Let us further examine the question of stationary resonance which can arise due to the perturbing momentum M_z^* (more precisely, due to its odd harmonics). Let us find odd r which satisfy the inequality $0 < r/s < \sqrt{3}$, with given s :

$s=1$	$r=1$
$s=3$	$r=1, 5$
$s=5$	$r=1, 3, 7$
$s=7$	$r=1, 3, 5, 9, 11$

With main resonance ($r, s = 1$), under the summation sign in system (24) stand terms with the indexes $n = 1, 3, 5, 7, \dots$. Let us suppose that k (and thus q , too) are sufficiently small and let us ignore terms in (28) which have $n \geq 3$. We then have

$$\begin{aligned} & \frac{q^{1/2}}{1+q} b_1 \cos(\alpha + \gamma_1) - \frac{2\alpha}{\pi} (E - k'^2 K) \\ k - k_0 = & \frac{\pi n^2}{4\alpha^2 k K (E - k'^2 K)} - \frac{q^{1/2} (1-q)}{(1+q)^3} b_1 \sin(\alpha + \gamma_1). \end{aligned} \quad (31)$$

System (31) can be solved numerically. Adopting $b_1 = 1$ for determinacy, we will find that system (31) has a solution where α from the interval $1 < \alpha \leq 1.07$. In studying resonance effects, we usually construct by the formula $\pi\alpha/2K = 2/s$ the so-called skeleton curve $k_0(\alpha)$, and also resonance curves $k^+(\alpha)$ and $k^-(\alpha)$. The co-

ordinates of curve $k^+(\alpha)$ are derived in solving the second equation /82 of (28) with a negative value $\sin(\kappa + \gamma_1)$; the coordinates of curve $k^-(\alpha)$ -- with a positive value. The functions $k_0(\alpha)$, $k^+(\alpha)$ and $k^-(\alpha)$ are incremental functions of α ; with an increase in α , the curves $k^+(\alpha)$ and $k^-(\alpha)$ approach each other and at some $\alpha = \alpha^*$ (α^* corresponds to the value $\cos(\kappa + \gamma_1) = 1$), the equalities $k^+(\alpha^*) = k^-(\alpha^*)$ are fulfilled. In our case $\alpha^* \approx 1.07$. When $\alpha = 1.06$, close to the right end of the interval of existence of the solution, we find that

$$\begin{array}{ll} k_0(\alpha) = 0,452 & \\ k^+(\alpha) = 0,457 & \theta^+ = 27^\circ 12' \\ k^-(\alpha) = 0,443 & \theta^- = 26^\circ 17' \end{array}$$

The largest value of q_1 corresponding to $k = 0.457$ is equal to $q \approx 0.016$, i.e., the disregard for terms with $q^{3/2} \approx 0.0021$ which took place above was fully justified. It is not hard to prove with the aid of numerical analysis that when $s \geq 3$, the first equation in (28) has no solution either at any $r/s < d < \sqrt{3}$, i.e., with real values of the parameters in the system, only main resonance is possible.

To study the stability of resonance conditions when $r, s = 1$, let us linearize system (24) in the neighborhood of resonance values k^* and κ^* , discard as before the terms with the indexes $n \geq 3$, and compose an equation in the variations

$$\begin{aligned} \frac{d(\delta k)}{du} &= -A_1 \sin(\kappa^* + \gamma_1) \delta \kappa, \\ \frac{d(\delta \kappa)}{du} &= \omega'(k_0) \delta k - B_1 \cos(\kappa^* + \gamma_1) \delta \kappa, \end{aligned} \quad (32)$$

where for brevity we designate that

$$A_1 = \frac{\varepsilon}{I'(k)} \cdot \frac{4q^{1/2}}{1+q} b_1, \quad B_1 = \frac{\varepsilon}{I'(k)} \cdot \frac{\pi^2}{kk'^2 K^2} \cdot \frac{q^{1/2}(1-q)}{1+q} b_1.$$

Let us note that always $A_1 > 0$, $B_1 > 0$, $\omega'(k_0) < 0$. Let us compose the characteristic equation of system (32);

$$\lambda^2 + B_1 \cos(\kappa + \gamma_1) \lambda + \omega(k_0) A_1 \sin(\kappa + \gamma_1) = 0$$

and an expression for its roots

$$\lambda_{1,2} = -\frac{1}{2} B_1 \cos(\kappa + \gamma_1) \pm \sqrt{\left[\frac{1}{2} B_1 \cos(\kappa + \gamma_1)\right]^2 - \omega'(k_0) A_1 \sin(\kappa + \gamma_1)}.$$

(33)

Since always $\cos(\kappa + \gamma_1) > 0$, the first summand in (33) is negative. If $\sin(\kappa + \gamma_1) > 0$, then λ_1, λ_2 are real and of different sign -- we have a saddle. If $\sin(\kappa + \gamma_1) < 0$, then according to the relationship of absolute values of the summands below the root sign, a particular point is either asymptotically stable or is a stable focus /83. Therefore, resonance conditions corresponding to the top resonance curve $k^+(\alpha)$ are stable; corresponding to the lower $k^-(\alpha)$ -- unstable.

5. In summing up the entire discussion, we come to the conclusion that in a system whose motion is described, under the assumptions taken, by equation (3), only main resonance is possible for the narrow band of values of α , lying to the right of one. Since in real systems like this usually $\alpha > 1$ ($\alpha \approx 1.6-1.6$)(1), resonance conditions can not arise in them. Motion will develop as an attenuating oscillation process; the stable conditions will be slight residual oscillations with respect to the position of stable equilibrium described by the value $\theta = 0$. The amplitude, phase shift and shift of center of these residual oscillations can be calculated using a linear theory and assuming θ to be small.

REFERENCES

1. Tipling, B.E., Merrick, V.K., Avtomaticheskoye upravleniye kosmicheskimi letatel'nymi apparatami [Automatic control of spacecraft], "Nauka" Publishers, 1968.
2. Sadv, Yu. A., "Periodic motion of a satellite with a magnetic damper in the plane of a round orbit," Kosmich. issled., 7, 1 (1969).
3. Chernous'ko, F. L., Zhurn. vychisl. matem. i matem. fiziki, 3, 1 (1963).
4. Zhuravskiy, A.M., Spravochnik po ellipticheskim funktsiyam [Guidebook on elliptical functions], Academy of Sciences of the USSR, 1941.
5. Beytmen, G., Erdeyn, A., Vysshiye transtsendentnyye funktsii. Ellipticheskiye i avtomorfnyye funktsii. Funktsii Lame i Mat'yo [Higher transcendental functions. Elliptical and automorphous functions. Lamé and Mathieu functions], "Nauka" Publishers, 1967.
6. Samoylova-Yakhontova, N.S., Tablitsy ellipticheskikh integralov [Elliptical integral tables], ONTI, 1935.
7. Philosophical Magazine, 7, 30 (1940), pp. 516-519.